

**Math 376** Homework Set #4 due Wednesday, May 1

Please do problems #1 - #4 on this sheet, and staple your book problems to this sheet. You must use proper notation and show the appropriate work to earn full credit.

1. (5 points) A container with capacity 20 liters initially holds 10 liters of water, with a concentration of 2 grams per liter. Suppose water with a concentration of  $5te^{-2t}$  grams per liter flows in at 2 liters per minute, while the well-mixed mixture flows out at 1 liter per minute. What is the concentration inside the container when it overflows?

2. (5 points) Find the solution of  $x' = -\frac{4}{t}x + \cos(t^5)$ ,  $x(1) = 0$ .

3. Consider the initial value problem  $x' = \frac{x^{\frac{2}{3}}}{\sqrt{t+1}} + t$ ,  $x(t_0) = x_0$ . For which values of  $t_0$  and  $x_0$  can you guarantee that there is a unique solution?

4. (5 points) For the IVP  $x' = -x + 3t$ ,  $x(0) = 1$ , use Euler's Method with  $h = \frac{1}{3}$  (by hand) and  $h = \frac{1}{30}$  (with *Mathematica*) to approximate  $x(1)$ . Plot both your approximations as well as the actual solution and calculate the error between your approximations and the actual solution at  $t = 1$ .

5. (5 points) Book Problems:

- Sect 2.3 #17
- Sect 2.4 #1, 5
- Sect 2.6 #5