Math 376

Homework Set #1 Solutions

2. (5 points) Using complete sentences, explain the difference in the meaning of the word "solution" in each of the following sentences: "1 is a solution of $\ln(x^2 - x + 1) = 0$", and "1 is a solution of $x' = \cos(x-1) - 1$".

   In the first, we mean the number 1 is a solution. In the second, we mean the constant function $x(t) = 1$ is a solution of the differential equation.

3. (5 points) Is $u(t) = e^{-t^2}$ a solution of the differential equation $u'' - 2tu' = 4t^2u$? Why or why not?

   If $u(t) = e^{-t^2}$, then $u'(t) = -2te^{-t^2}$, and $u''(t) = -2e^{-t^2} + 4t^2e^{-t^2}$. Therefore, we will have

$$u'' - 2tu' = -2e^{-t^2} + 4t^2e^{-t^2} + 4t^2e^{-t^2} = (8t^2 - 2)e^{-t^2},$$

   and

$$4t^2u = 4t^2e^{-t^2}.$$

   Since $(8t^2 - 2)$ is not the same function as $4t^2$, we see that $u(t) = e^{-t^2}$ is not a solution of the differential equation.

4. (5 points) Evaluate $\displaystyle\int \frac{2}{x^2 - 6x + 8}dx$.

   We use partial fraction decomposition, since $x^2 - 6x + 8 = (x-4)(x-2)$. Thus, we look for constants A and B such that $\frac{2}{x^2-6x+8} = \frac{A}{x-4} + \frac{B}{x-2}$. Multiplying both sides by $x^2 - 6x + 8$, we see that $2 = A(x-2) + B(x-4)$. Now, if we take $x = 2$, we get $2 = B(-2)$, and so $B = -1$. Next, taking $x = 4$, we see that $2 = 2A$, and so $A = 1$. Therefore, we know that $\frac{2}{x^2-6x+8} = \frac{1}{x-4} - \frac{1}{x-2}$. Therefore,

$$\int \frac{2}{x^2 - 6x + 8}dx = \int \frac{1}{x-4} - \frac{1}{x-2}\ dx = \ln|x-4| - \ln|x-2| + C = \ln\left|\frac{x-4}{x-2}\right| + C.$$