

## Math 260

Quiz #5, due Friday, May 3

To earn full credit, you must show your relevant work and use correct notation!

1. Suppose  $S$  is a non-empty set, and suppose  $P(x, y)$  is a predicate for  $x, y \in S$ . Outline proofs for both  $\exists x \in S \forall y \in S P(x, y)$  and  $\forall y \in S \exists x \in S P(x, y)$ .

2. Prove that for any integers  $a, b$  and  $c$ , if  $a|b$  and  $b|c$ , then  $a|c$ . (You might take a look at Quiz #4.)

3. Consider the theorem: “if  $x \bmod 7 = 1$ , then  $(x^2 - 3x) \bmod 7 = 5$ .”

(a) Give an outline for a proof of this theorem.

(b) Prove this theorem.

4. Consider the theorem: “whenever  $x \in \mathbb{Z}$  and  $d \in \mathbb{Z}^+$ , if  $x \bmod d = 1$ , then  $d \mid (x^2 - x)$ .”

(a) Give a format for a proof of this theorem.

(b) Prove this theorem.