**Math 260**

Review/Outline for Exam #2

Exam #2 will be Wednesday, May 8. You will be allowed two $8\frac{1}{2}$" $\times$ 11" sheets of paper with handwritten notes on both sides.

Sect 1.5 **Rules of Inference**: You should know what a syllogism is, as well as what it means for a syllogism to be valid. Given a syllogism, you should be able to translate it into a symbolic form and identify whether or not it is valid or invalid, and be able to explain why.

Sect 1.8 **The Conditional Redux**: You should read this section, and be aware that there are a lot of ways to express $p \to q$ in mathematical english.

Sect 2.1 **The Direct Proof, the Universal Proof and the Existential Proof**: You should know the basic format of each of the following types of statements:

1. Conditionals - format of a proof of $P \to Q$: Assume $P$ is true. [Show $Q$ is true.]
2. Univeral Statements - format of a proof of $\forall x \in S(P(x))$: Suppose that $x \in S$ is arbitrary. [Show $P(x)$ is true.]
3. Existential Statements - format of a proof of $\exists x \in S(P(x))$: [Construct a candidate for $x \in S$.] [Show that $P(x)$ is true.]

In addition to these basic forms, you should know how to set up the format of a proof of statements such as $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z}^+\ \big(x < 0 \to y > x\big)$ or $\forall a \in \mathbb{R}^+\ \exists \delta \in \mathbb{R}^+\ \forall x \in \mathbb{R}\big(|x-1| < \delta \to |\sqrt{x} - 1| < a\big)$.

Sect 2.2 **Divisibility**: We defined what it meant to say that $a|b$ for integers $a$ and $b$. You should know this definition, as well as be able to work with it to show various statements involving divisibility.

Sect 2.3 **The Division Algorithm**: We stated the division algorithm, and used it to show that any integer is either even or odd, and no integer can be both. We also defined what it meant to say that $x \bmod d = y$. You should know this definition, and be able to use it to prove statements involving relations between integers.

Sect 2.4 **The Contrapositive Proof**: We discussed the format of a proof using the contrapositive, as well as gave some idea as to when to use it. Notice that you can only use contrapositive to prove a conditional $P \to Q$. You need to explicit as to what the contrapositive is. The format of a proof using the contrapositive is: We use the contrapositive. Therefore, suppose $\neg Q$. [Show $\neg P$.]

Section 2.5 **Proof by Cases**: You should know how to use cases to prove statements, as well as know the format of proof using cases. You should review some of the examples from this section. Remember to be explicit as to what your cases are!

Section 2.6/2.7 **Proof by Contradiction**: The format of a proof by contradiction is: Suppose the negation of what you're trying to prove. Then, show a contradiction. The proof by contradiction is useful when what you're trying to show is stated in the negative, as in $\neg Q$. You should be very explicit about what you are assuming when you do a proof by contradiction!

## Practice Problems

1. Consider "there exists an integer $y$ such that for all integers $x$, if $x$ is negative, then $y > x$."
   (a) Translate this into symbols.
   (b) Give a format of a proof.
   (c) What is the truth value of this statement? Why?

2. (a) Give a format for a proof that if $a \bmod d = r$ and $b \bmod d = r$, then $d|(a-b)$.
   (b) Prove that if $a \bmod d = r$ and $b \bmod d = r$, then $d|(a-b)$.

3. Prove or disprove: for any $x \in \mathbb{Z}$ and any $d \in \mathbb{Z}^+$, if $x \bmod d = 2$, then $(x+3) \bmod d = 5$.

4. Consider the statement: for any integers $a, b, c$, if $a|b$ and $a|c$, then for any integers $k_1, k_2$, $a|(k_1 b + k_2 c)$.
   (a) Give an outline of a proof.
   (b) Prove this statement.

5. Consider the statement: "for any integers $x, y$, if $x \bmod 3 = 2$ and $y \bmod 3 = 2$, then $xy \bmod 3 = 1$."

(a) Give an outline of a proof.

(b) Prove this statement

6. Consider the statement: "for any integer $x$, if $x \bmod 4 = 1$, then $(x^2 - 3x) \bmod 4 = 2$."

    (a) Give a format of a proof.

    (b) Prove this statement.

7. Consider the statement: "for any integer $x$, if $4 \nmid (x^2 - x)$, then $4 \nmid x$.

    (a) Give a format of a proof by contrapositive.

    (b) Prove this statement.

8. Suppose that $x \bmod d = r_1$ and $y \bmod d = r_2$. Show that if $(x + y) \bmod d = z$, then $(r_1 + r_2) \bmod d = z$.

9. Prove that for any positive real numbers $x$ and $y$, if $x + y < 10$, then $x < 2$ or $y < 8$.

10. Consider the statement "for any integer $x$, if $x \bmod 3 \neq 0$, then at least one of $x + 2$ or $x + 4$ is divisible by 3".

    (a) Give a format for a proof of this statement using cases. (Hint: what are the only possible values of $x \bmod 3$?)

    (b) Prove this statement.

11. Consider the statement "if $3|x^2$, then $3|x$".

    (a) Give a format for a proof by contrapositive that uses cases.

    (b) Prove this statement.

12. Show that $\sqrt{8}$ is irrational.

13. Show that $\log_2 3$ is irrational.