Math 260

Review/Outline for Exam #1

The first mid-term exam is Wednesday, April 17, and you are allowed a single $8\frac{1}{2}" \times 11"$ sheet of paper with handwritten notes on both sides.

Sect 1.1 **Statements, Logical Operators and Truth Tables**: We discussed what a statement is, as well as various ways to combine statements with the symbols $\wedge, \vee, \neg$ as well as the conditional $\rightarrow$. Given statements, you should know how to determine the truth value of compound statements based upon the truth value of the component parts. In addition, you should be able to construct truth tables.

Sect 1.2 **Biconditionals, Logical Equivalencies and Implications**: You should know the truth value of a biconditional based upon its component parts. In addition, you should be able to explain the distinction between logical equivalencies and biconditionals, as well as the distinction between a conditional and an implication. You should be able to use the various logical equivalencies to show two statements are equivalent. It is useful to have the logical equivalencies from your book on your sheet of notes!

Sect 1.3 **Sets and Predicates**: You should be familiar with and be able to use set notation. We also discussed predicates and their truth sets. You should know the difference between a statement and predicate. Given a predicate and a domain for that predicate, you should be able to describe the truth set using the roster method. For example, consider $P(x)$ given by "$x \in \mathbb{N} \rightarrow 1 \leq 2x - 1 \leq 6$" with domain $\mathbb{R}$. What is the truth set of $P(x)$? Are there any integers in the truth set of $P(x)$? What about non-integers?

Sect 1.4 **Quantifiers**: You should know the meaning of the quantifiers $\exists$ and $\forall$, as well as be able to determine the truth value of statements involving them. You will be asked questions about the truth value of statements with mixed quantifiers, so you should know how the statement $\forall x \in S\ \exists y \in S\ P(x, y)$ differs from $\exists y \in S\ \forall x \in S\ P(x, y)$.

Sect 1.5 **Rules of Inference**: You should know what a syllogism is, as well as what it means for a syllogism to be valid. Given a syllogism, you should be able to translate it into a symbolic form and identify whether or not it is valid or invalid, and be able to explain why.

Sect 1.6 **Negations**: We discussed the negations of various statements. In particular, you should know how to negate the quantifiers $\forall$ and $\exists$.

Sect 1.7 **How $\forall$ and $\exists$ Cooperate with $\vee$ and $\wedge$**: You should know how $\forall$ and $\exists$ distribute (or not!) over the logical connectors $\vee$ and $\wedge$. It's worthwhile to make up your own examples to help you remember how these work!

Sect 1.8 **The Conditional Redux**: You should read this section, and be aware that there are a lot of ways to express $p \to q$ in mathematical english.

Sect 2.1 **The Direct Proof, the Universal Proof and the Existential Proof**: You should know the basic format of each of the following types of statements:

1. Conditionals - format of a proof of $P \to Q$: Assume P is true. [Show Q is true.]
2. Univeral Statements - format of a proof of $\forall x \in S(P(x))$: Suppose that $x \in S$ is arbitrary. [Show $P(x)$ is true.]
3. Existential Statements - format of a proof of $\exists x \in S(P(x))$: [Construct a candidate for $x \in S$.] [Show that $P(x)$ is true.]

In addition to these basic forms, you should know how to set up the format of a proof of a statement such as $\forall x \in \mathbb{Z}\ \exists y \in \mathbb{Z}^+\ \big(x < 0 \to y > x\big)$.

Problems and Examples: There are a lot of problems in the book that weren't assigned. The examples in the book can also be helpful.