

Math 260

Practice for Mid-Term Exam #1

- Are the following statements or not? If so, explain how you would test the truth value.
 - Lisa is married to Ralph.
 - There are 10^{100} molecules of oxygen in the room.
 - A car weighs more.
 - Which way to Walla Walla?
- Construct a truth table for $(P \wedge Q) \vee (P \rightarrow Q)$.
- Is $(P \rightarrow R) \wedge (Q \rightarrow R)$ logically equivalent to $(P \wedge Q) \rightarrow R$? Why or why not?
- Translate the following into symbols:
 - Anybody who likes “The Simpsons” likes “Futurama” as well.
 - Anybody who doesn’t like coffee also doesn’t like chocolate.
 - There is a exactly one solution of $x^2 + 2x + 1 = 0$.
- Let $T(x, y)$ be the predicate “ x teaches y ”, where $S := \{\text{people}\}$. Translate the following into everyday english and say (if possible!) whether or not the statement is true or false.
 - $\exists x \in S \forall y \in S T(x, y)$
 - $\forall y \in S \exists x \in S T(x, y)$
 - $\exists x \in S T(\text{Dr. Bisgard}, x)$
 - $\forall x \in S (T(x, \text{Dale}) \rightarrow T(x, \text{Mike}))$
- Let $S := \mathbb{Z}$. Are the following true or false? Why?
 - $\exists y \in S \forall x \in S (x \cdot y \text{ is divisible by } 2)$
 - $\forall x \in S \exists y \in S (2x + y = 0)$.
 - $\forall y \in S \exists x \in S (2x + y = 0)$.
 - $\exists x \in S \forall y \in S (2x + y = 0)$.
 - $\exists y \in S \forall x \in S (2x + y = 0)$.

7. What is the difference between a statement and a predicate?
8. What is the difference between a conditional and an implication?
9. Let $P(x)$ be the predicate $x \in \mathbb{Z}^+ \rightarrow x - 1 = 0$ with domain $S = \mathbb{R}^+$.
 - (a) Determine the truth value of $P(\pi)$.
 - (b) Determine the truth value of $P(4)$.
 - (c) What is the truth set of $P(x)$?
 - (d) What is the truth set of $\neg P(x)$?
10. Write a format for a proof of $\forall x \in \mathbb{R} \exists y \in \mathbb{Z}^+ \left(x > 0 \rightarrow \frac{1}{y} < x \right)$. It isn't necessary to prove this statement!
11. Consider the syllogism "Every Canadian loves hockey. Tim isn't Canadian. Therefore, Tim doesn't love hockey."
 - (a) Translate this into symbols.
 - (b) Is this syllogism valid or invalid? Why?