

Due Monday May 19 by 3 pm

**Instructions.** You are encouraged to work together although each student must turn in his/her own write-up.

1. Let  $(X, \rho)$  be a compact metric space and suppose  $\{f_j\}$  and  $\{g_j\}$  are uniformly convergent sequences in  $C(X)$ . Prove that the sequence  $\{f_j g_j\}$  converges uniformly.

2. Suppose  $f \in C([0, 1])$  and  $\int_0^1 f(x)x^n dx = 0, \forall n \in \mathbb{N}$ . Prove that  $f(x) = 0, \forall x \in [0, 1]$ . Hint: First show

that  $\int_0^1 f(x)p(x)dx = 0$  for any polynomial  $p$ . Then use the Weierstrass Approximation Theorem to

argue that  $\int_0^1 (f(x))^2 dx = 0$ .