Advanced Analysis III | Spring 2008 | HW #5

Due Wednesday May 7 by 3 pm

Instructions. You are encouraged to work together although each student must turn in his/her own write-up.

1. Describe convergent sequences in $(\mathbb{R}, \rho)$ where ρ is the discrete metric.

2. Let $C^1([0,1])$ be the set of all continuously differentiable functions on the interval $[0,1]$. For $f, g \in C^1([0,1])$ define
$$d(f,g) = \sup_{x \in [0,1]} |f'(x) - g'(x)| .$$
Is ρ a metric? Why or why not?

3. The Bolzano-Weierstrass Theorem tells us that in $\mathbb{R}$ a bounded infinite set must have a limit point. Show by example that the analogous statement is false in an arbitrary metric space.

4. Let (X, ρ) be a metric space. Let $f : X \to \mathbb{R}$ be a function. Prove that f is continuous if and only if $f^{-1}(U)$ is open whenever $U \subseteq \mathbb{R}$ is open.