Advanced Analysis III Spring 2008 HW #4

Due Friday April 25 by 3 pm

Instructions. You are encouraged to work together although each student must turn in his/her own write-up.

1. Prove Proposition 9.1: A sequence of functions $\{f_j\}_{j=1}^{\infty}$ is uniformly Cauchy on a domain S if and only if the sequence $\{f_j\}_{j=1}^{\infty}$ converges uniformly to a limit function f on the domain S.

2. Suppose the sequence $\{f_j\}_{j=1}^{\infty}$ converges uniformly to a function f on a domain S. If each f_j is bounded on S prove that f is also bounded on S.

3. Suppose the sequence $\{f_j\}_{j=1}^{\infty}$ converges pointwise to a function f on a domain S. Let $M_j = \sup\{|f_j(x) - f(x)| : x \in S\}$. Prove that $\{f_j\}_{j=1}^{\infty}$ converges uniformly to f if and only if $\{M_j\}_{j=1}^{\infty}$ converges to 0.