Advanced Analysis III | Spring 2008 | HW #3

Due Wednesday April 16 by 3 pm

Instructions. You are encouraged to work together although each student must turn in his/her own write-up.

1. Suppose f is continuous on $[a, b]$. Prove that there exists a $c \in [a, b]$ such that $\int_a^b f(x)\, dx = f(c)(b-a)$. This result is called the *Mean Value Theorem for Integrals*.

2. Let $f(x) = x^2 + 1$.

   (a) Find the value of $c \in [2, 6]$ such that $\int_2^6 f(x)\, dx = f(c)(6-2)$.

   (b) Give an example where the mean value theorem for integrals fails if f is not continuous.

3. Suppose f is continuous and nonnegative on $[a, b]$. Let $M = \sup\{f(x) : x \in [a, b]\}$. Prove that

$$\lim_{n\to\infty} \left[\int_0^1 [f(x)]^n dx\right]^{\frac{1}{n}} = M$$

4. Let A be a dense subset of $[a, b]$. If f is integrable on $[a, b]$ and $f(x) = 0$ for all $x \in A$, prove that $\int_a^b f(x)\, dx = 0$.