Asset cash flows: Payments that a company receives; e.g. investment returns, insurance premiums.

Liability cash flows: Payments that a company is required to make; e.g. insurance claims, benefit payments, debt.

Surplus: Present value of the assets minus the present value of the liabilities.
\[
\text{Surplus} = \text{PV}\{\text{Assets}\} - \text{PV}\{\text{Liabilities}\}
\]
Both PV’s are interest sensitive.

Immunization: the act of protecting (or immunizing) a surplus position from negative change due to interest rate change. An immunized position is that a change in the interest rate does not decrease the surplus.

Yield curve: the graph of a bond’s yield against the bond’s time until maturity. A yield curve gives you the interest rates for various lengths of investment periods.

Redington immunization:

- **Assumption**: Assume all the yield curves are flat (yields do not vary by maturity—short term and long term yields/interest rates are the same, which is often not the case in practice) and an interest rate change is a parallel shift in the flat yield curve. For example, if one-year short term interest rate changes from 5% to 6%, then 20-year long term rate also changes from 5% to 6%.
- **Three immunization conditions**: if met will immunize the surplus from negative change given small changes in the interest rate used to discount the assets and liabilities.
  1. \( \text{PV}\{\text{assets}\} = \text{PV}\{\text{liabilities}\} \)

     It’s okay if \( \text{PV}\{\text{assets}\} \geq \text{PV}\{\text{liabilities}\} \)

  2. **Duration of assets = Duration of liabilities**

     Recall that the **modified duration** measures the percentage value change (of assets as well as liabilities!!) as the yield shift by 100 basis points.

     By investing in the asset portfolio having the same duration as the liabilities, we have the approximately the same (as the liabilities) percentage increase in the value of an asset that occurs when the yield on
the asset shifts down by 100 basis points and the yield on the liability shifts down by 100 basis points, vice versa.

3. **Convexity of assets > Convexity of liabilities**

\[
\frac{P(y + \Delta y) - P(y)}{P(y)} \approx (-Duration)(\Delta y) + \frac{(Convexity)(\Delta y)^2}{2}
\]

**Positive convexity** associates an increase in the price of assets (as well as liabilities) after an interest rate shift (to either direction, up or down). The larger the convexity is, the larger increase in the assets/liabilities value. In a immunization position, we hope assets value increase is higher than the liabilities’.

- The three conditions for immunization may be satisfied at a time point but not constantly since the duration and convexity of assets and liabilities are affected differently by the changes of the yields.

- **Rebalance**: By periodically selling and buying some assets, a portfolio manager can maintain a position that satisfies the three immunization conditions.

- **Redington immunization** protects the surplus for small shifts in a flat yield curve. Under certain circumstances, the surplus is also protected for large shifts in a flat yield curve and such position is called **fully immunized**. A fully immunized position has positive surplus regardless of the magnitude of the interest rate change. Redington immunization may not protect the surplus from a nonparallel shift in the yield curve. As a result, there is a **revised condition 3** to mitigate the this interest risk by bringing the asset cash flows closer to the liability cash flows.

**Revised condition 3.**

**Convexity of assets** \( \rightarrow \) **Convexity of liabilities**

Examples 7.5-7.7 on pages 368-377.

**Full immunization** protects the surplus for all shifts in a flat yield curve. The surplus is protected for large shifts in a flat yield curve:

\[
PV[\text{assets}] \geq PV[\text{liabilities}] \text{ for all flat } i > 0.
\]

In calculation:

\[
\sum A_i \cdot v^i \geq \sum L_i \cdot v^i \text{ for any } i > 0.
\]
Three conditions: for full immunization of a single liability cash flow under flat yield curve assumption.

1. \( \text{PV}[\text{assets}] = \text{PV}[\text{liabilities}] \quad \text{(same!)} \)  
   It’s fine with \( \text{PV}[\text{assets}] \geq \text{PV}[\text{liabilities}] \)

2. Duration of assets = Duration of liabilities (same!)

3. The asset cash flows occur before and after the single liability cash flow. (new!)

Example 7.8 on page 379-380.

Remark: 1. If a company wants to fully immunize a position consisting of multiple liability cash flows, it can do so by allocating the assets (or portions of the assets) to each liability cash flow, so that the conditions for full immunization are satisfied for each liability cash flow and its designated assets. 2. A fully immunized portfolio always satisfies the conditions for Redington immunization.

Dedication: also known as cash flow matching, matches the asset and liability cash flows exactly. For each liability payment there is an equal asset payment made at the same time. A dedicated portfolio of assets is constructed to fund a set of known liabilities such as in large pension plans where liability payments are predictable with a great deal of accuracy. Advantage for dedication is no need to rebalance the asset portfolio. Disadvantage is the lower yield due to limited bonds can be purchased to match the cash flows.

HW Chapter 7 Due on Friday, April 15th.