Spokes or Discs?

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Introduction

It is well known that disc wheels and standard spoked wheels exhibit different performance characteristics on the race track, but as yet no reliable means exist to determine which is superior for a given set of conditions.

We create a model that, taking the properties of wheel, cyclist, and course into account, may provide a definitive answer to the question, "Which wheel should I use today?" The model provides detailed output on wheel performance and can produce a chart indicating which wheel will provide optimal performance for a given environmental and physical conditions.

We use laws of physics, plus data from various Web sites and published sources, then numerical methods to obtain solutions from the model.

We demonstrate the use of the model's output on a sample course. Roughly speaking, *standard spoked wheels perform better on steep climbs and trailing winds, while disc wheels are better in most other cases*.

We did some validation of the model for stability, sensitivity, and realism. We also generalised it to allow for a third type of wheel, to provide a more realistic representation of the choice facing the professional cyclist today.

A major difficulty was obtaining reliable data; sources differed or even contradicted one another. The range of the data that we could find was insufficient, jeopardizing the accuracy of our results.

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Analysis of the Problem

Consider the system of a cyclist and the racing cycle. The cyclist provides the energy to drive the bicycle against the forces of drag (from contact with air), friction (from contact between wheels and ground), and gravity (which opposes progress up a slope). Furthermore, when accelerating, the cyclist must provide the energy to set the wheels rotating, due to their moment of inertia.

The primary problem is to determine, for a given set of conditions, which type of rear wheel is the most effective. "Effectiveness" is what a specific rider desires from the equipment. We assume that the rider desires to complete the course in as short a time as possible, or with the least possible energy expenditure. These definitions are closely linked: A rider who expends more energy to maintain a certain speed will soon tire and will therefore have a lower maintainable speed.

The differences between standard 32-spoke wheels and disc wheels lie in weight and in their aerodynamic properties. Given the right wind conditions (which we investigate), the disc wheels should allow air to pass the cyclist/cycle combination with less turbulence, that is, less drag. However, disc wheels weigh more, which affects the amount of power required to move the wheels up a slope and to begin rotating the wheels from rest (such as when accelerating).

To examine which type of wheel performs the best under which conditions, we need to determine which wheel allows the greatest speed given specific conditions or, equivalently, which wheel requires the least power to drive.

The greatest difficulty is that air resistance is a function of speed while speed is a function of air resistance. The model needs to utilise numerical methods to calculate the speed that a rider can maintain with the given parameters.

There are other factors to consider, too. Disc wheels are not very stable in gusty wind conditions, since they provide a far greater surface area to cross-winds; with a greater moment of inertia, they accelerate more slowly; and their greater weight may provide more grip on wet roads.

Assumptions and Hypothesis

We investigated the performance of the wheel types noted in **Table 1**. The wheel data do not conform to any specific make or model but are typical.

Table 1.

Types of wheels.

Type	Standard 32-spoke	Aero wheel (trispoke)	Solid disc wheel
Diameter (m)	0.7	0.7	0.7
Mass (kg)	0.8	1.0	1.3

We assume that the cyclist uses a standard spoke wheel in the front and either a disc wheel or standard 32-spoked wheel at the back. We also briefly examine aero wheels, which are not solid but more aerodynamic than standard spoked wheels. We refer to the three types as standard, aero, and disc.

We assume that the rider and cycle frame (excluding wheels) exhibit the same drag for all wind directions.

Table 2.
Symbols used.

Symbol	Unit	Definition
A	m^2	area of rider/bicycle exposed to wind
c_a	dimensionless	variable coefficient of axial air resistance for specific wheel
c_{rr}	dimensionless	constant of rolling resistance
c_w	dimensionless	constant of air resistance
D	m^2	reference area of wheel
F_{ad}	Newton (N)	axial air resistance (against the cyclist's direction of motion)
F^*_{ad}	Newton (N)	axial air resistance on a bicycle with box-rimmed spoked wheels in a headwind
F_g	Newton (N)	effect of gravity on the cyclist
F_{rr}	Newton (N)	rolling resistance
g	m/s^2	gravitational constant
M	kg	mass of cyclist and cycle
P	Watt (W)	rider's effective power output
v_{bg}	m/s	speed of the bike relative to the ground
v_{wb}	m/s	speed of the wind relative to the bike
v_{wg}	m/s	speed of the wind relative to the ground
α	degree	angle of the rise
β	degree	yaw angle, the angle between the direction opposing bicycle motion and perceived wind (A relative headwind has a yaw of $0°$.)
γ	degree	angle between wind direction (v_{wg}) and direction of motion (A straight-on headwind has $\gamma = 180°$.)
ψ	percent	grade of hill, the sine of α, the angle of the rise
ρ	kg/m^3	air density

Forces at Work

For a bicycle moving at a constant velocity, there are three significant retarding forces (**Figure 1**):

rolling resistance, due to contact between the tires and road;

gravitational resistance, if the road is sloped; and

air resistance, usually the largest of the three.

When accelerating, the rider also uses energy to overcome translational and rotational inertia, although the model does not take these into account.

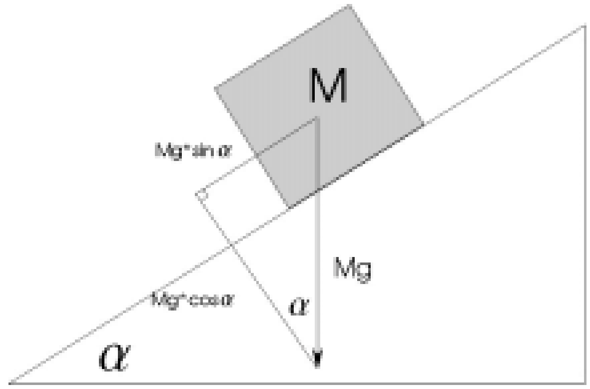

Figure 1. Diagram of forces.

The forces of rolling resistance and gravity are as follows:

$$\begin{aligned} F_{rr} &= c_{rr} \cdot (\text{normal force}) \\ &= c_{rr} M g \cos \alpha \\ &= c_{rr} M g \cos \arcsin \psi \\ &= c_{rr} M g \sqrt{1-\psi^2}, \\ F_g &= M g \sin \alpha = M g \psi. \end{aligned}$$

Calculating the axial drag force is more complicated. The air resistance is $f = \frac{1}{2}\rho \cdot c_w A v^2$, where v is the speed of the air relative to the object. Since we assume that the area of the rider/frame exposed to the wind is constant, we have (neglecting the additional drag on the wheels caused by yaw and type of wheel)

$$F_{ad}^* = \tfrac{1}{2}\rho \cdot A v_{wb}^2 \cos \beta \qquad \text{(axial component)}.$$

Observe the sketches in **Figure 2**. From them we derive that

$$\begin{aligned} v_{wb}^2 &= v_{wg(\text{axial})}^2 + v_{wg(\text{side})}^2 \\ &= \big(v_{bg} + v_{wg}\cos(180^\circ - \gamma)\big)^2 + \big(v_{wg}\sin(180^\circ - \gamma)\big)^2 \\ &= (v_{bg} - v_{wg}\cos\gamma)^2 + (v_{wg}\sin\gamma)^2 \\ &= {v_{bg}}^2 - 2 v_{bg} v_{wg} \cos\gamma + v_{wg}^2. \end{aligned}$$

Also,

$$\beta = \arctan\left(\frac{v_{wg(\text{side})}}{v_{wg(\text{axial})}}\right) = \arctan\left(\frac{v_{wg}\sin\gamma}{v_{bg} - v_w \cos\gamma}\right).$$

The axial air drag on the rear wheel [Tew and Sayers 1999] is

$$F_{ad(\text{wheel})} = 0.75 \cdot \tfrac{1}{2}\rho \cdot c_a D \cdot v_{wb}^2;$$

the 0.75 is because a rear wheel experiences 75% of the drag of a wheel in free air, due to interference of the gear cluster, frame, cyclist's legs, and so forth.

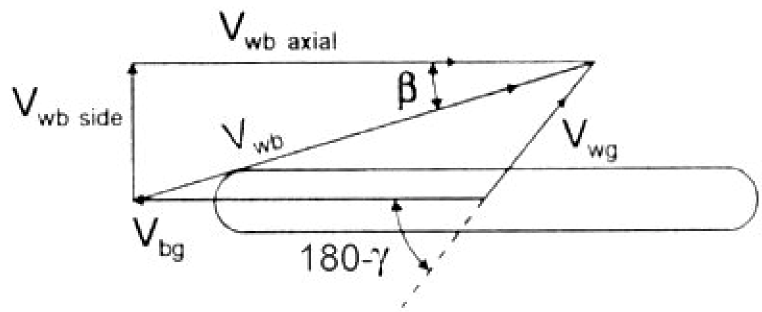

Figure 2a. Wind speed relative to wheel.

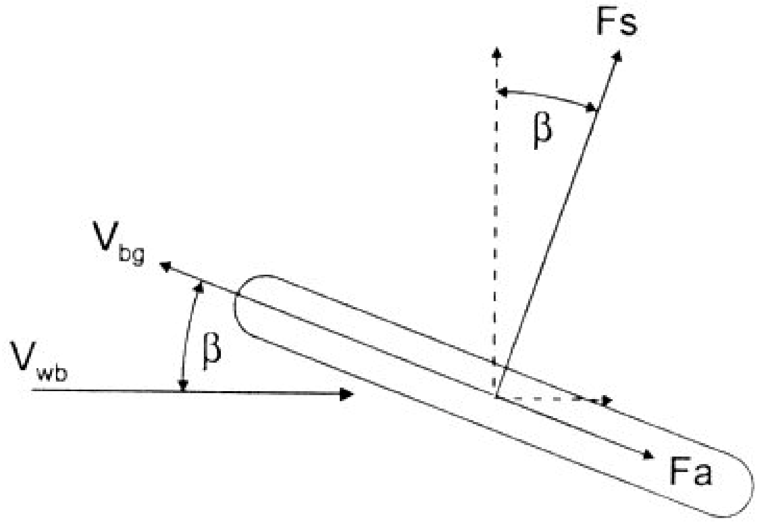

Figure 2b. Forces on wheel.

For the three basic types of wheel, Tew and Sayers [1999] give typical curves of the axial drag coefficient c_a vs. yaw angle ($0° \leq \beta \leq 30°$); this interval accounts for the majority of conditions experienced by a rider. We approximate these curves by straight lines (a close match).

The curves for different relative wind speeds are very much alike for the standard wheel and the aero wheel. The disc wheel, however, shows major variation for different relative wind speeds.

Since the axial drag coefficient must be zero at $\beta = 90°$, and by observing the shape of the curves, we extrapolated to larger yaw angles using a sine-shaped curve through $c_a = 0$ at $\beta = 90°$, with an appropriate scaling to ensure continuity. Without wind-tunnel testing, the accuracy cannot be guaranteed.

Comparing the percentage of power dissipated by drag on one wheel (according to the model) with the data of Tew and Sayers [1999], we found a high degree of agreement. Typically, 1% to 10% (depending on wheel type) of the power is dissipated by drag on the wheels.

From $F_{ad(\text{wheel})}$ we subtracted the drag experienced by a normal (box-rimmed) wheel under headwind, since it was already taken account in F^*_{ad}.

The axial air drag on the bicycle is thus given by

$$F_{ad} = F^*_{ad} + F_{ad(\text{wheel})} = \tfrac{1}{2}\rho v_{wb}^2 [c_w A \cos\beta + 0.75(c_a - 0.06)D],$$

where 0.06 is the coefficient of axial drag for a normal wheel in a headwind. The $\cos\beta$ gives the component in the direction of motion of the cyclist.

Calculating Results for a Typical Rider

For data, we used standard values [Analytic Cycling 2001] for a road racer near sea level in normal atmospheric conditions:

$$\begin{aligned} M &= 80 \text{ kg}, & g &= 9.81 \text{ m/s}^2, \\ c_{rr} &= 0.004, & c_w &= 0.5, \\ A &= 0.5 \text{ m}^2, & D &= 0.38 \text{ m}^2 \quad \text{(for a 700 mm wheel)}, \\ \rho &= 1.226 \text{ kg/m}^3 & & \text{(could be changed to incorporate altitude)}. \end{aligned}$$

We calculated that, to maintain a speed of 45 km/h on a level road (as in the problem description), the rider must deliver 340 W of effective pedaling power.

The Computer Program

We wrote a computer program in Pascal that calculates the speed that the rider can sustain for a given v_{wg}, γ, and ψ. It does this by trying a speed and determining the wattage necessary to sustain the speed. If the wattage is too high, the speed is lowered; otherwise, the speed is increased. Every time the solution point is crossed, the step size is reduced. The process is carried out until the wattage used is within a tolerance 0.01 W to P.

To take into account the effect of drag on different types of wheels, our program does the following:

1. The wind direction, wind speed relative to the ground, and slope of the road (γ, v_{wg}, and ψ) are provided as inputs.
2. The program tries a value for v_{bg}.
3. F_{rr} and F_g are calculated.
4. From γ, v_{wg}, ψ, and v_{bg}, we calculate v_{wb} and β.
5. From v_{wb} and β, we calculate F_{ad}.
6. We calculate the wattage by using the formula $P = (F_{rr} + F_g + F_{ad})v_{bg}$.
7. We compare the calculated value of P to the known value of 340 W.

8. We try a new value for v_{bg}, depending on whether the wattage required for the previous value of v_{bg} was higher or lower than the available 340 W.

9. We repeat this process from Step 3 until the maximum maintainable speed is determined.

Since the wheel that requires the least power in a set of circumstances also enables the highest speed, we used our program to vary the speed of the wind and show which wheel is best for the circumstances. **Figure 3** shows a screen shot. The dark colour represents blue and the light colour red. Each of the 11 horizontal strips represents a road gradient, ranging from 0 at the top to 0.1 at the bottom in 0.01 increments.

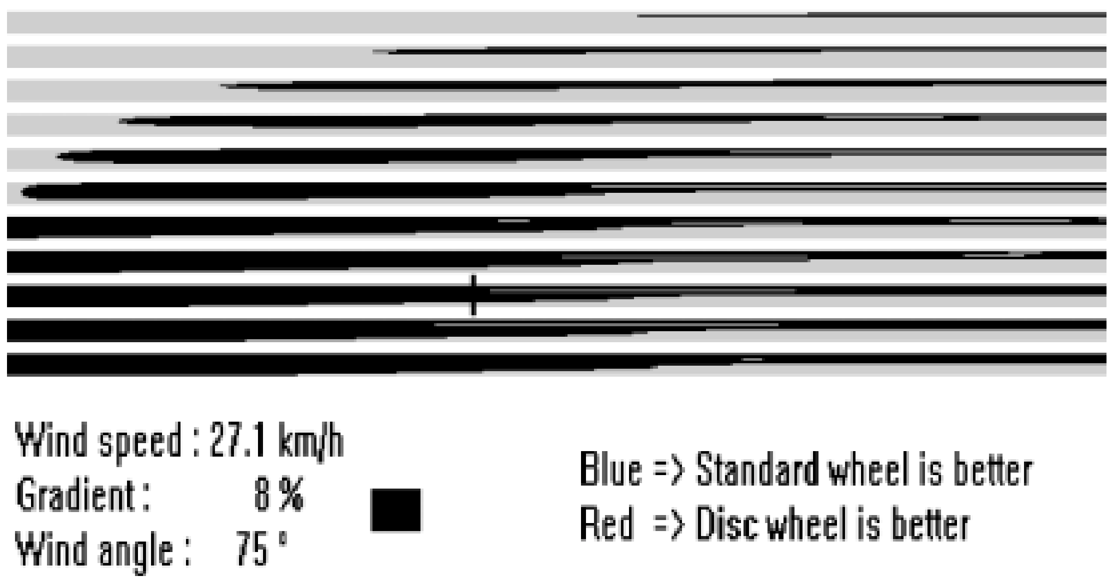

Figure 3. Screen shot from program.

The horizontal axis is wind speed, from 0 km/h at the left to 63.9 km/h at the right in 0.1 km/h increments. The vertical axis of each bar is the wind angle (relative to track), from 0° to 180° in 15° increments.

We have a very compact representation showing the transition wind speeds for a range of wind angles and road gradients. The user is provided a crosshair to move over any point on the graph. The colour of a pixel indicates the better wheel to use for the corresponding gradient, wind speed, and wind angle.

To generate a table of transition speeds (**Table 3**), we read off the points at which transitions occur. This might seem cumbersome, but developing an algorithm to find the transition points is very difficult, since the number of transitions is not known beforehand and the functions exhibit irregular behaviour.

To use the table, look up the particular entry corresponding to the road grade and the angle that the wind makes with the forward direction of the bike. An entry of S means that a standard wheel performs better for all wind speeds, a D indicates that a disc wheel is better at all wind speeds.

Table 3.

Which wheel to use, as a function of road grade and wind angle.

Road grade	Angle of wind (in degrees) relative to bike's direction 0	15	30	45
<0	D	D	D	D
0	D	D46.5S	D36.6S	D
0.01	D	D27.8S	D22.6S	D21.3S47.3D
0.02	D44.5S	D16.5S	D13.3S	D12.3S53.8D
0.03	D24.2S	D9.4S47.6D54.5S	D7.3S56.6D	D6.5S
0.04	D12.3S	D4.8S	D3.5S42.0D	D3.0S
0.05	D4.2S	S	S33.9D	S
0.06	S	S63.4D	S54.8D61.8S	S38.6D46.2S
0.07	S	S	S55.6D59.1S	S32.3D46.5S
0.08	S	S	S	S28.0D45.8S
0.09	S	S	S	S24.9D44.8S
0.10	S	S	S	S

Road grade	Angle of wind (in degrees) relative to bike's direction 60	75	90	105
<0	D	D	D	D
0	D	D	D	D
0.01	D22.1S33.6D	D	D	D
0.02	D12.7S39.1D	D14.5S32.0D	D	D
0.03	D6.4S43.2D	D6.9S35.1D	D8.5S30.6D	D14.2S23.8D
0.04	D2.9S47.9D	D2.8S37.2D	D3.2S33.4D	D3.8S29.6D
0.05	S	S40.0D	S34.7D	S32.4D
0.06	S	S42.8D	S35.7D	S34.2D
0.07	S	S46.6D	S37.5D	S35.5D
0.08	S	S	S39.2D	S36.4D
0.09	S	S	S40.6D	S37.2D
0.10	S	S	S42.1D	S37.8D

Road grade	Angle of wind (in degrees) relative to bike's direction 120	135	150	165	180
<0	D	D	D	D	D
0	D	D	D	D	D
0.01	D	D	D	D	D
0.02	D	D	D	D	D
0.03	D	D	D	D	D
0.04	D6.1S22.1D	D	D	D	D
0.05	S28.0D	S17.4D	D	D	D
0.06	S31.1D	S23.7D	S15.4D	S8.3D	S1.7D
0.07	S33.3D	S27.9D	S19.8D	S13.7D	S6.5D
0.08	S34.9D	S30.6D	S23.0D	S17.5D	S10.5D
0.09	S36.3D	S32.5D	S25.5D	S20.2D	S13.8D
0.10	S37.4D	S34.1D	S27.6D	S22.6D	S16.8D

The other entries can be decoded as follows: A number between two letter entries indicates at which wind speed a transition occurs; the first letter indicates which wheel is most efficient at lower speeds, and the second number which wheel is best at higher speeds. For example, S28.0D indicates that standard wheels are better at speeds below 28.0 km/h. An entry of D6.1S22.1D indicates that the standard wheel performs better at speeds between 6.1 and 22.1 km/h, while the disc wheel performs better at all other wind speeds.

The table applies for wind speeds up to 64 km/h. Strong winds are very rare and a disc wheel will cause major stability problems in these conditions.

As an aside, we created graphs comparing standard, aero, and disc wheels simultaneously and allowed for negative gradients as well. The aero wheel dominated in most conditions.

Applying the Table to a Sample Course

We designed a simple time-trial course. The map of the course and a view of the elevation are given in **Figure 4**. The course consists of four different segments, with each turning point labelled with a letter. The data for each point are in **Table 4**.

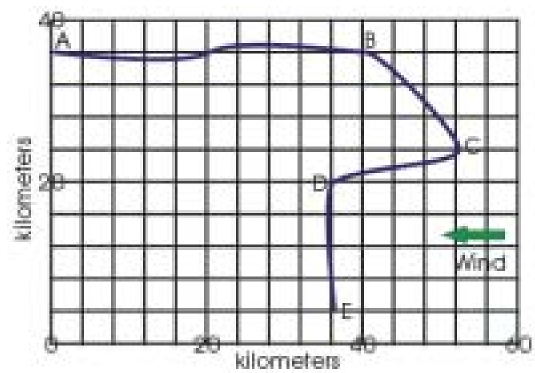

Figure 4a.

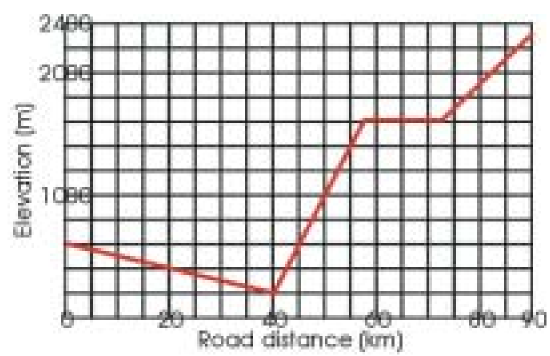

Figure 4b.

Table 4.
Details of the sample course.

Point	Map coordinates (km, km)	Elevation (m)
A (Start)	(0,36)	600
B	(40,36)	200
C	(52,24)	1600
D	(36,20)	1580
E (Finish)	(36,4)	2350

Assume that the wind is blowing at 25 km/h in the direction shown on the map. For each segment, we compute the gradient and the angle of the segment with the wind by trigonometry. The length of each segment is slightly longer than the straight-line distance, because the road is not perfectly straight.

We look at **Table 3** to determine the best wheel for each section of the course. For instance, in the second section, the gradient is 0.08 and the angle 135°; according to the table, the standard wheel is better at a wind speed below 30.6 km/h, so at 25 km/h, a standard wheel is better for this section. We fill in the other entries in a similar manner (**Table 5**).

Table 5.
Best wheel for each section of the sample course.

Section	Distance (km)	Wind angle (°)	Gradient	Best wheel
AB	40.8	180	−0.01	Disc
BC	17.5	135	0.08	Standard
CD	16.7	14	0.00	Disc
DE	16.2	90	0.05	Standard

The disc wheel and the standard wheel both win in two segments. However, the disc wheel wins over 58 km of the course, while the standard wheel wins over only 33 km. Thus, the table advises that the cyclist use the disc wheel.

Getting More Refined Results

For each section, we calculated the expected speed for the rider with each wheel. We add the times for the individual sections to obtain an estimate of the total time, obtaining **Table 6**. The table shows two interesting results:

- The disc wheel beats the standard wheel by about 50 s. This is consistent with the result obtained earlier in this section.
- The aero wheel is almost 2 min faster than the disc wheel!

Table 6.
Total time for sample course for each wheel.

		Standard wheel		Aero wheel		Disc wheel	
Section	Length (km)	Speed (km/h)	Time taken (h:min:sec)	Speed (km/h)	Time taken (h:min:sec)	Speed (km/h)	Time taken (h:min:sec)
AB	40.8	32.73	1:14:47.6	33.27	1:13:34.8	33.46	1:13:09.7
BC	17.5	12.56	1:23:35.9	12.68	1:22:48.5	12.47	1:24:12.1
CD	16.7	59.52	0:16:50.1	60.13	0:16:39.8	59.97	0:16:42.5
DE	16.2	19.72	0:49:17.4	19.94	0:48:44.8	19.58	0:49:38.6
Total	91.2		3:44:31		3:41:48		3:43:43

Validating the Model

Sensitivity Analysis

Does the table generated for one rider with a specific set of physical attributes apply to another rider, and if not, can the model easily be adjusted?

To determine whether the same table could be used for different riders, we varied one of the rider's parameters, either power output, mass, or cross-sectional surface area, while keeping the others constant. In these analyses we found that

- Changing any one or any combination of the parameters P, A, or M does not affect the basic pattern but slightly distorts (shifts, scales, or skews) it.
- Every rider-cycle combination would need its own chart for determining which wheel to use at which speed.

Other Validation

We compared our model's output to data available at Analytic Cycling [2001], which provides interactive forms. Our model's output matched their output almost exactly for all the different combinations of input parameters that we used. Unfortunately, this site does not make provision for wind speed or angle, so this part of our model could not be compared.

We tested the model with a completely different set of parameters approximating a very powerful sports car ($P = 300$ kW, $C_d = 0.3$, $A = 2.3$ m^2, $M = 1100$ kg). We kept the other parameters the same. Our model predicted a top speed of 320 km/h on a level road, which seemed very realistic.

Error Analysis

We were concerned about the disc wheel "islands" that showed up in our graphical output at wind speeds of 40–50 km/h, wind angles of 30°–60°, and higher gradients (see **Figure 3**). They probably are due to the peculiar behaviour of disc wheels in crosswinds. Since we extrapolated the drag coefficient function, we have no way of knowing whether this strange behaviour is realistic or not.

Model Strengths

If a rider can obtain reasonably accurate course data (something that is not difficult at all), then the rider can determine exactly what type of rear wheel to use for a race by referencing this information to a chart or computer.

The model has many parameters (air density, coefficients, rider mass, etc.) that can be adjusted to account for various situations.

It is easy to extend the model to include the front wheel of the bicycle.

Model Weaknesses

When the angle of the wind with the cyclist changes, there are a number of factors that influence the amount of drag that the cyclist experiences. The most obvious is the changing surface area; a cyclist from the side presents a far larger surface area to the wind than a cyclist heading into the wind.

Less obvious, but still a large contributing factor, is that the drag coefficient is a function of the shape of the object. A cyclist from the side is far less streamlined and has a higher drag coefficient.

Both these factors are extremely difficult to model. The cross-sectional surface area of a complex three-dimensional shape could be the subject of a paper on its own, and determining the drag coefficient of the same complex shape would require empirical tests.

As a result, we ignored these effects and assume that the drag coefficient and cross-sectional surface area of the rider are the same for all directions. Strong cross-winds would cause too much rider instability to even consider using disc wheels, so it would not really be necessary to investigate drag in such cases.

We also ignored the effects of the energy needed to overcome the rotational and translational inertia for each wheel. We cannot comment on the effect of these forces, since we did not have time to implement these effects.

The model is only as accurate as the data used, and much of the available data on wheel drag is, at best, dubious. Many wheel manufacturers exaggerate the performance of their brands, while rival manufacturers quickly denounce their findings. Further, few data were available for yaw angles greater than 30°, and we were forced to extrapolate, leading to a high degree of uncertainty.

Using the table requires specific information about rider characteristics, some of which may be difficult to obtain, such as the surface area of rider and bicycle.

Conclusion

Our model provides a good means for making an informed decision as to which wheel to use in a particular situation, but it needs more accurate data and refinement.

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